Linear Oscillations of a Drop in Uniform Alternating Electric Fields

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Oscillations of a conducting drop immersed in a dielectric fluid in an alternating electric field were modeled to understand the enhancement of transport processes by the electric field. Numerical solutions for oscillation amplitude, velocity distribution, resonant frequency and streamlines were obtained. The effects of viscosity and density on the resonant frequency and the velocity distribution were investigated. It was found that the resonant frequency of viscous fluids was always smaller than the free oscillation frequency of the same droplet. The predicted scanning frequency response curve and the streamlines agree well with the experimental observations.

Introduction

Applications of an electric force in direct contact heat transfer processes or liquid extraction can result in a better energy efficiency than the conventional agitation methods (Thornton, 1968). For liquid systems with insulating continuous phases and conducting dispersed phases, an imposed electric field will disturb the interface by interacting with induced electric charge and thus enhance mass or heat transfer rate. In a static field, the electric force can reduce drop size, increase drop velocity, and therefore increase the interfacial area and the transport coefficient. An alternating electric field can also reduce the drop size effectively (Kawalski and Ziolkowski, 1981). Experiments on direct contact heat transfer showed that an alternating electric field with a proper frequency was more efficient than a static field in enhancing the heat transfer coefficient (Kaji et al., 1980, 1985). The experiments also showed that the enhancement of the heat transfer coefficient was related directly to the drop oscillations induced by the electric force. To understand the effects of the electric fields on the transport processes, we need knowledge of the hydrodynamics of the electrically forced drop oscillations.

Studies on free oscillations of a drop (Miller and Scriven, 1968; Prosperetti, 1980; Marston, 1980) have revealed that a drop oscillates at its characteristic frequencies no matter how the oscillations are excited. For viscous fluids, the oscillation amplitudes decay gradually due to viscous dissipation. Drop oscillations under the sustained action of an external alternating force are different. The oscillation frequencies are the same as those of the external forces (Lamb, 1945), but the amplitudes are functions of the external forces as well as properties of the fluid system. A quasi-steady external force maintains the amplitudes constant; therefore, the decay factors are zero. Torza et al. (1971) and Sozou (1972) have investigated oscillations of an uncharged drop in alternating electric fields. The attention of their work was paid to the effects of electric properties (resistivity and dielectric constant) on the drop deformation and the flow patterns. They did not discuss the effects of the hydrodynamic properties, which are more essential to the study of the transport processes. Drop oscillations driven by acoustic waves were studied by Marston (1980). Explicit solutions for oscillation amplitudes and phase shift angles were obtained for low-viscosity systems. For solutions in a wide viscosity range, numerical methods must be used.

In this article, we present a model for the hydrodynamics of small-amplitude oscillations of a conducting drop immersed in a dielectric fluid in an alternating electric field. Expressions for velocity distributions are obtained analytically, and boundary condition equations are solved numerically. Resonant frequencies are predicted and the effects of viscosity and density on flow fields are discussed.

Model

Consider a charged fluid sphere immersed in another immiscible fluid. Both fluids are assumed incompressible and Newtonian, and properties in each phase are uniform. The interface is presumed to be free from any contamination by surfactants. It is also assumed that the effect of gravity on drop deformation can be neglected ($gR^2\Delta\rho/\sigma$ is small). Only small-amplitude oscillations are considered; therefore, the nonlinear term in the Navier-Stokes equation can be neglected

(Levich, 1962). It is further assumed that electric field far from the drop is uniform, the drop phase is conductive and the continuous-phase dielectric so that electric equilibrium can be reached instantly and charge loss negligible. This study is limited to the axisymmetric flow, which has been frequently observed in experiments. Spherical coordinates (r, θ, ϕ) will be used and the origin is at the drop center. $\theta = 0$ is taken as the symmetry axis (the electric field is in this direction). The momentum equation is:

$$\frac{\partial v}{\partial t} = \nu \nabla^2 v - \frac{1}{\rho} \nabla p \quad (1)$$

The continuity equation for an incompressible fluid is:

$$\nabla \cdot v = 0 \quad (2)$$

By taking the divergence of the momentum equation and subtracting the continuity equation, the following pressure equation is obtained (Chandrasekhar, 1961):

$$\nabla^2 p = 0 \quad (3)$$

The boundary conditions include: continuity of the normal and tangential velocities at the interface; normal stress balance at the interface, $\tau_{ri} + \tau_s = \tau_{ro}$; continuity of the tangential stress at the interface; normal velocity at the interface matching with the displacement rate of the interface, $\partial \zeta / \partial t = v_r|_{r=\zeta}$; and finite pressure and velocities at the drop center as well as at infinity. When the drop center moves, the origin of the coordinates moves with it. Consequently, it seems that the surrounding fluid moves while the drop position keeps unchanged. In the normal stress balance, τ_s is the surface stress including the effects of surface tension and surface charge.

$$\tau_s = \sigma \left(\frac{1}{R_1} + \frac{1}{R_2} \right) - \tau_e \quad (4)$$

where σ is interfacial tension, R_1 and R_2 are principal radii of curvature of the drop surface, and τ_e is stress produced by electric charge on the interface.

For free oscillations of a drop with net charge on its surface, the primary effect of net charge on the oscillations is induced by redistribution of the charge on deformed drop surface. A first-order perturbation model (weighted in terms of deformation amplitude) is necessary to describe this charge redistribution (Rayleigh, 1882; Hendricks and Schneider, 1963). When an external alternating electric field exists, direct interaction between the charge and the field is the primary effect on drop motion and oscillations. This effect can be described by a zero-order perturbation model. The effect of charge redistribution on the deformed surface becomes secondary. Under ordinary conditions, the secondary effect is much smaller than the primary effect. Only the primary effect will be considered in this article; therefore, a zero-order perturbation model is used. Theoretically, the boundary conditions should be satisfied on the deformed interface. For a zero-order perturbation model, the boundary conditions will be satisfied on the undeformed spherical interface, and the normal and tangential components of vectors will be replaced by r and θ components, respectively.

In zero-order perturbation, the electric charge distribution for a slightly deformed drop is approximated by that on a spherical conductor (Reitz, 1967),

$$q = q_{\text{net}} + 3\epsilon E \cos\theta \quad (5)$$

where q_{net} is the average net charge density, ϵ is the permittivity of the surrounding fluid, E is field strength, and θ is the polar angle measured from the positive direction of the electric field. For an alternating electric field, $E = E^o \cos(\beta_e t)$, where E^o is amplitude and β_e is frequency of the electric field. The normal stress generated by the charge is:

$$\tau_e = \frac{q^2}{2\epsilon} = \left(\frac{q_{\text{net}}^2}{2\epsilon} + \frac{3}{2}\epsilon E^2 \right) + 3 q_{\text{net}} E^o \cos(\beta_e t) P_1 + \frac{3}{2}\epsilon E^{o2}[\cos(2\beta_e t) + 1] P_2 \quad (6)$$

where $P_n = P_n(\cos\theta)$ is a nth-order Legendre polynomial. There is no electric tangential stress on a conducting drop surface (Taylor, 1966). The terms in the first parenthesis represent average pressure produced by the net and induced charge. For an incompressible fluid, this average pressure alters only the pressure distribution, but does not contribute to any drop movement. The second and third terms contain only first- and second-order Legendre functions, respectively. According to the linear oscillation theory (Lamb, 1945), the above electric forces will affect only the first mode (linear translation) and the second mode (prolate-oblate oscillation) motions, respectively. Oscillation frequencies are β_e and $2\beta_e$ for the first and second modes, respectively. The time-independent part in the last term will cause a static prolate deformation.

If higher-order corrections were included in the model, secondary effects of drop deformation would induce oscillations of some other modes, but the amplitudes of those oscillations would be much smaller than the primary motions considered here, as long as the assumption of small amplitude is satisfied.

The solution procedure for the above equations is similar to that described by Chandrasekhar (1961). The expressions of the solution are in complex forms, which involve half-order Bessel and Hankel functions.

$$p_i = p_i^o + \rho_i \sum_{n=1}^{2} a_{1n} \exp(-\omega_n t) r^n P_n \quad (7)$$

$$p_o = p_o^o + \rho_o \sum_{n=1}^{2} a_{2n} \exp(-\omega_n t) r^{-n-1} P_n \quad (8)$$

$$v_{ri} = \sum_{n=1}^{2} \exp(-\omega_n t) \left[\frac{a_{1n} n r^{n-1}}{\omega_n} + \frac{a_{3n}}{r^{3/2}} J_{n+1/2}(x_i) \right] P_n \quad (9)$$

$$v_{ro} = \sum_{n=1}^{2} \exp(-\omega_n t) \left[-\frac{a_{2n}(n+1)}{\omega_n r^{n+2}} + \frac{a_{4n}}{r^{3/2}} H_{n+1/2}(x_o) \right] P_n + \frac{a_{51}}{\omega_1} \exp(-\omega_1 t) P_1 \quad (10)$$

$$v_{\theta i}=\sum_{n=1}^{2}\exp(-\omega_n t)\left\{\frac{a_{1n}r^{n-1}}{\omega_n}+\frac{a_{3n}}{r^{3/2}}\left[\frac{J_{n+1/2}(x_i)}{n}-\frac{x_i J_{n+(3/2)}(x_i)}{n(n+1)}\right]\right\}\frac{dP_n}{d\theta} \quad (11)$$

$$v_{\theta o}=\sum_{n=1}^{2}\exp(-\omega_n t)\left\{\frac{a_{2n}}{\omega_n r^{n+2}}+\frac{a_{4n}}{r^{3/2}}\left[\frac{H_{n+1/2}(x_o)}{n}-\frac{x_o H_{n+(3/2)}(x_o)}{n(n+1)}\right]\right\}\frac{dP_n}{d\theta}+\frac{a_{51}}{\omega_1}\exp(-\omega_1 t)\frac{dP_1}{d\theta} \quad (12)$$

$$\zeta=R+[a_{52}\exp(-\omega_2 t)+b_2]P_2 \quad (13)$$

Where p and p^o are pressure and its time-independent part, respectively; v is velocity; subscripts i and o denote inner and outer fluids; subscripts r and θ denote r and θ components, respectively; n is mode number; $\omega_n=\pm in\beta_e$, is a complex number corresponding to the oscillation frequency of nth mode; $x=\sqrt{\omega_n/\nu r}$; $a_{kn}(k=1,2,3,4,5)$ and b_2 are unknown constants; $J_{n+1/2}$ and $H_{n+1/2}$ are half-order Bessel and Hankel functions of the first kind, respectively; ζ represents deformed drop surface; and R is the undeformed spherical drop radius. The last terms in Eqs. 10 and 12 represent fluid velocity far from the drop relative to the drop center.

Because the problem is linear, we can use the complex form in the boundary conditions and find out the unknown constants numerically, and then take the real parts of the results at the end of the calculations. Substituting the above expressions into the boundary conditions and after some algebraic manipulation, a system of linear algebraic equations for each oscillation mode can be obtained. The equations can be solved numerically with the Gauss elimination method to get values of the unknown constants.

It is more efficient to present the results in dimensionless form. The variables are nondimensionalized as follows:

$$r^*=r/R,\ v^*=v/(\beta_e R),\ a^*=|a_{5n}|/R,$$
$$p^*=p/(\rho_i\beta_e^2R^2),\ \psi^*=\psi/(\beta_e R^3)$$

where r, v, a, p, and ψ are radial coordinate, velocity, amplitude, pressure, and stream function, respectively; the superscript * denotes dimensionless variables. Dimensionless groups that affect the dimensionless solution of the second-mode oscillation are:

$$e_i,\ e_o,\ \beta^*,\ \rho_o/\rho_i,\ f$$

where $e_i=\nu_i\sqrt{\rho_i/\sigma R}$ and $e_o=\nu_o\sqrt{\rho_o/\sigma R}$ may be regarded as dimensionless viscosities; $\beta^*=\beta_e\sqrt{\rho_i R^3/\sigma}$, is a dimensionless frequency; ρ_o/ρ_i is the density ratio; and $f=3\epsilon E^{o2}R/2\sigma$ is the ratio of electric stress to the surface tension stress. $f=0.1$ is used in this work. The time-independent part of the normal stress balance gives the solution for b_2: $(b_2/R)=f/4$.

Results and Discussion

Although first-mode motion is also oscillatory and may play an important role in enhancing transport processes, it involves only linear translation, but not shape deformation. Solution to the first mode can be obtained by the same method used for the second mode. The following discussions emphasize the second-mode shape oscillation.

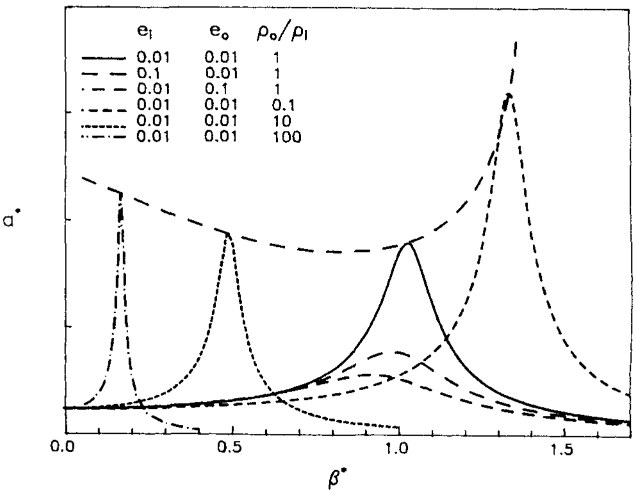

Figure 1. Predicted scanning frequency response curves for second-mode oscillation.

The line passing the peaks of the response curves indicates the effect of density ratio on the resonant amplitude for $e_i=e_o=0.01$.

Frequency scanning response and resonant frequency

Amplitudes of forced oscillations are functions of the electric field and properties of the fluid system. The linear theory predicts that the amplitude is proportional to the external force. Effects of other parameters are complicated. By measuring the oscillation amplitude while varying the frequency of the force, a scanning frequency response curve can be obtained. Figure 1 shows calculated response curves. Since the dimensionless amplitude a^* is proportional to f and only its relative significance is needed here, its scale is arbitrary. All the curves in

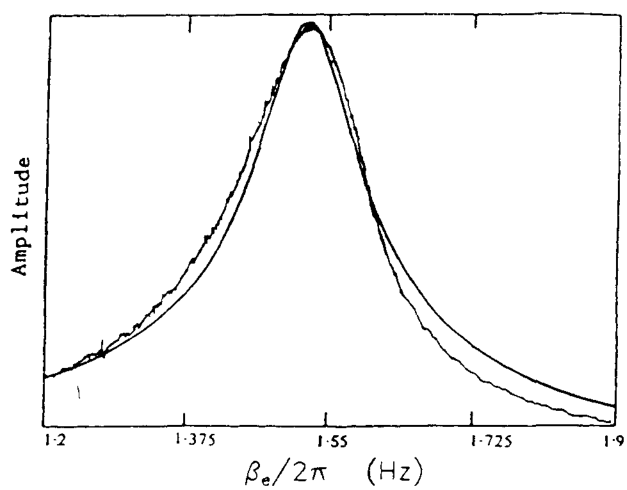

Figure 2. Predicted vs. observed (Trinh et al., 1982) second-mode scanning frequency response curves for a silicone/CCl_4 drop immersed in distilled water.

The smooth curve is predicted. Drop volume = 1.5 cm^3, $\rho_i=\rho_o=990$ kg/m^3, $\nu_i=3.2\times10^{-6}$ m^2/s, $\nu_o=1.01\times10^{-6}$ m^2/s.

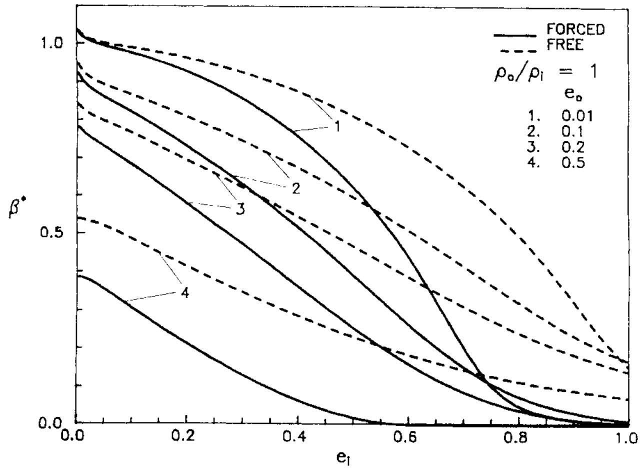

Figure 3. Resonant frequency for second-mode: forced oscillation vs. free oscillation.

Figure 1 have the same value when β^* approaches zero. At this point, the drop is in a quasi-equilibrium state and the only parameter that affects a^* is f. Therefore, a constant f gives a constant a^*. When β^* approaches ∞, a^* approaches zero. In between, the curve may pass through a peak, which indicates resonance. The shape of the peak is determined by e_i, e_o, and ρ_o/ρ_i. When e_i or e_o increases, the response curve becomes flattened and the dimensionless resonant frequency decreases. When ρ_o/ρ_i increases, the dimensionless resonant frequency decreases, but the resonant amplitude first decreases and then increases. Figure 2 compares a predicted scanning frequency response curve with that measured by Trinh et al. (1982) for a drop driven by an acoustic force. The predicted resonant frequency is tuned to coincide with the measured one by adjusting the interfacial tension (calculated at 0.029 N/m compared to 0.035-0.04 N/m given in their article). The shapes of the curves are similar, indicating that electric force and acoustic force have similar effects on drop deformation.

The resonant frequencies can be obtained from the frequency response curves. Figure 3 compares the resonant frequencies calculated by this model with the free oscillation frequencies

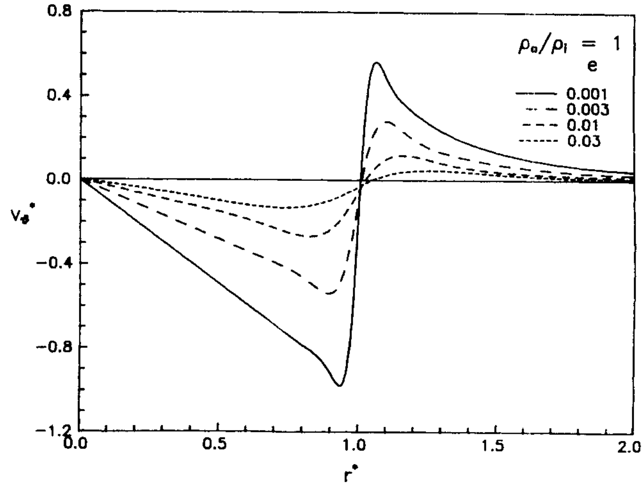

Figure 5. Tangential velocity distribution for a drop undergoing second-mode oscillation under the same conditions as those of Figure 4.

(Prosperetti, 1980). When calculating dimensionless frequency for free oscillation, the oscillation frequency should be divided by the mode number to be consistent with the definition for the forced oscillation. The effects of viscosities on free and resonant frequencies are similar. As viscosities increase, the frequencies decrease. For inviscid fluids, the resonant frequency coincides with the natural frequency. For viscous fluids, the resonant frequency is always smaller than the free oscillation frequency. The larger the viscosities, the greater the difference between them.

Velocity distribution

Variations of velocities in the θ direction are described by Legendre functions. Only the variations in the r direction will be discussed. For convenience, $\theta = 0$ is chosen for v_r^* and $\theta = \pi/4$ for v_θ^*, because in these directions the velocities are maximum. Other factors that affect the velocities are e_o, e_i, ρ_o/ρ_i, β^*, and time. It is clear from the velocity expressions that the

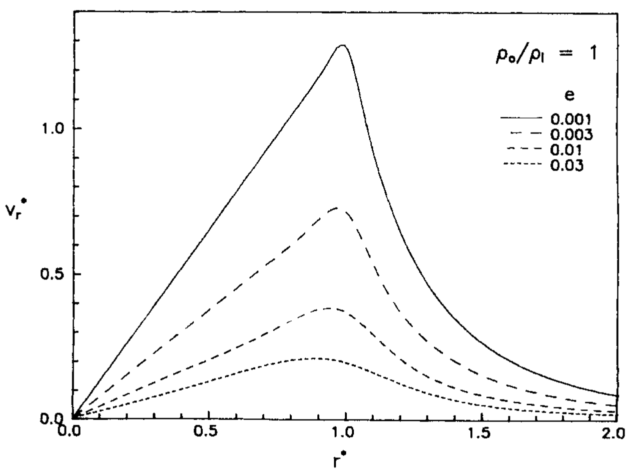

Figure 4. Radial velocity distribution for a drop undergoing second-mode oscillation with $e_i = e_o = e$ as a parameter.

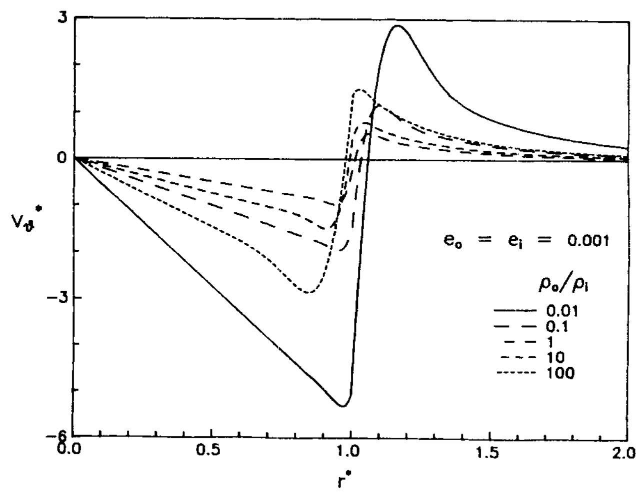

Figure 6. Tangential velocity distribution for a drop undergoing second-mode oscillation with ρ_o/ρ_i as a parameter.

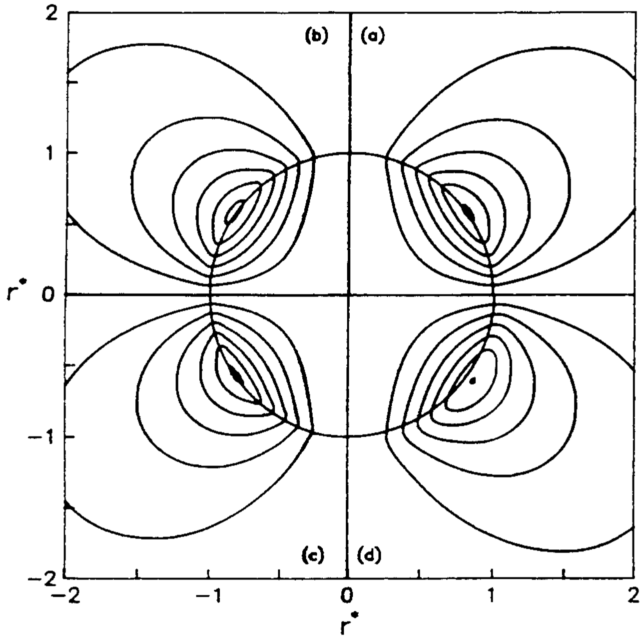

Figure 7. Streamlines for a drop undergoing second-mode oscillation.

(a) $e_i = e_o = 0.001$, $\rho_o/\rho_i = 1$; (b) $e_i = 0.01$, $e_o = 0.001$, $\rho_o/\rho_i = 1$; (c) $e_i = e_o = 0.001$, $\rho_o/\rho_i = 100$; (d) $e_i = e_o = 0.001$, $\rho_o/\rho_i = 0.01$.

velocities change with time periodically. The effect of β^* on the velocities is similar to that on the amplitude. Therefore, these two factors will not be discussed further. In the following, the resonant frequency and the time for maximum radial velocity at the interface are used. Figures 4 and 5 show the radial and tangential velocity profiles with $e_i = e_o = e$ as a parameter. The velocities decrease with increasing e. When e is small, the drop-phase velocities are proportional to the radius except near the interface, and the tangential velocity has very large gradients on both sides of the interface. The gradients increase with decreasing e, but no slip of velocity occurs as long as e is not zero. Consequently, the viscous dissipation is concentrated in a thin boundary layer near the interface for small-viscosity fluids, as has been predicted by Miller and Scriven (1968) for a free oscillating drop. If e_i or e_o changes individually, the velocity gradient in the corresponding side of the interface will be mostly affected.

Effect of density ratio on the tangential velocity is illustrated in Figure 6 for $e_i = e_o = 0.001$. Of the profiles shown in the figure, $\rho_o/\rho_i = 1$ gives the smallest velocity. The velocity profiles shift outward if $\rho_o/\rho_i < 1$, and inward if $\rho_o/\rho_i > 1$. In either of the cases, the larger tangential velocity gradient is always in the less dense side of the interface.

Streamlines

Since the flow field of the oscillating drop is axisymmetric, the stream functions can be easily obtained from the integration of the velocity expressions. Flow patterns for mode 2 are illustrated in Figure 7. The effects of viscosity and density ratio are illustrated in different quadrants. The circle in the middle indicates the interface of the drop. In all the conditions, the internal and external streamlines meet at the interface and form closed cycles. During oscillation, the fluids move back and forth once in a period along the streamlines. The centers of the cycles indicate the positions of stagnant rings where the radial and tangential velocities are zero. The θ position of the rings are determined by $v_r = 0$, which gives $\theta = \cos^{-1}\sqrt{1/3}$. $v_\theta = 0$ determines the radial position of the rings, which turns out to be a function of many parameters. When the densities are the same (a and b), the stagnant rings move to the less viscous side of the interface, while when the viscous effects are the same (a, c, and d), the rings move to the less dense side of the interface. The internal streamlines observed by Trinh et al. (1982) qualitatively agree with the predicted results.

Conclusions

The proposed oscillation model for a drop in an alternating electric field is able to predict velocity field, scanning frequency response curve, resonant frequency, and streamlines. The predicted scanning frequency response curve and the streamlines are in good agreement with experimental observations. It is found that the resonant frequency of the forced oscillation is equal to the free oscillation frequency for inviscid fluids, but always smaller than the free oscillation frequency for viscous fluids. The viscosities and the density ratio have significant influence on the velocity profiles.

The enhancement of a transport process by an alternating electric field depends on how much the system is disturbed. Operation at a resonant frequency is desirable because it produces maximum disturbance for the same energy input. The predicted scanning frequency response curves can give best operation frequency range for any fluid system. The velocity distribution is very useful for the modeling of a mass or heat transfer process.

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Notation

- a^* = dimensionless oscillation amplitude
- a_{kn} = constants in expressions of solutions at $k = 1, 2, 3, 4, 5$
- b_2 = second-mode static deformation
- E, E^o = alternating electric field strength, amplitude of E, V/m
- e = $\nu\sqrt{\rho/\sigma R}$
- f = $3\epsilon E^{o2}R/(2\sigma)$
- $H_{n+1/2}$ = half-order Hankel function
- i = $\sqrt{-1}$
- $J_{n+1/2}$ = half-order Bessel function
- n = oscillation mode number
- P_n = Legendre function
- p, p^o = pressure, static part of pressure, N/m^2
- q, q_{net} = surface charge density, net surface charge density, Coul/m^2
- R = equivalent spherical drop radius, m
- R_1, R_2 = principal radii of curvature of drop surface, m
- r = radial coordinate, m
- t = time, s
- v = velocity, m/s
- x = $r\sqrt{\omega/\nu}$

Greek letters

- β_e = angular frequency of electric field, s^{-1}
- ϵ = permittivity of the continuous phase, Farad/m

ζ = radial position of deformed drop surface, m
θ = coordinate
ν = kinematic viscosity, m^2/s
ρ = density, kg/m^3
σ = interfacial tension, N/m
τ = stress, N/m^2
ψ = streamfunction, m^3/s
ω = $\pm in\beta_e$

Subscripts

e = electric field
i = inside the drop
n = 1, 2, oscillation mode number
o = outside the drop
r = radial direction
s = on the interface
θ = θ direction

Superscripts

* = dimensionless variables

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